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BOOK REVIEWS

Scaling. By Grigory Isaakovich Barenblatt. Cambridge University Press, 2003. 186 pp. ISBN 0512 826578. £50.00 (hardback); ISBN 0521 533945. £19.99 (paperback).

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This book will become a classic. It concerns the subject of scaling, where one quantity depends on other quantities through a power-law relation.

Barenblatt's delightful book though, is more than a just an introduction to scaling: it can also be read as a philosophy of mathematical modelling. The writing is witty, insightful, and sometimes moving. Every time you read the book, you return refreshed and inspired. Nevertheless the book is on the whole difficult and challenging.

After a useful preface by Alexandre Chorin which guides the reader well, the author embarks on a concise introduction to dimensional analysis. The introduction relates an exciting anecdote about G. I. Taylor's derivation of a similarity solution modelling the very intense shock from a nuclear explosion. The first chapter then provides a more formal treatment of dimensional analysis. As a teaching aid and quick reference guide the first chapter alone justifies acquisition of the book. Several short examples, such as a proof of Pythagoras' theorem and an analysis of rowboat performance as a function of the number of oarsmen illustrate the subject perfectly.

A central theme is the application of dimensional analysis to finding exact similarity solutions of partial differential equations. The basic and well-known idea is to set up an initial-value problem on a partial differential equation, and to look for a solution as a function of spatial coordinates, time and other defining parameters. As explained in the second chapter, one then selects a primary subset of the parameters which are dimensionally independent, such that combinations of primary parameters can be used to non-dimensionalize the other parameters. The Buckingham-Π theorem (as stated in the first chapter) can then be used to eliminate the primary parameters. Subsequent substitution into the equation – if one is lucky – reduces it to one of lower dimensionality, often an ordinary differential equation. Sometimes the reduced equation can be solved analytically. When one or more dimensionless groups is small, greater simplification follows from neglect of small parameters. When applicable, this method is referred to as *complete similarity* or *similarity of the first kind*. Expressions of the form $\Pi = f(\Pi_1, \Pi_2, \ldots, \Pi_l)$ are obtained where the Π are the dimensionless groups. The great power of dimensional analysis to reduce the number of physical experiments required to explore relationships between quantities is often exploited by experimentalists but, as Barenblatt indicates, not always. Failure to remember to perform such an analysis results in much wasted human effort. One might add to this that dimensional analysis is a useful technique in computational experiments, too.

The third chapter goes deeper. Many applied mathematicians might think that the existence of a similarity solution is a happy accident. Faced with a new problem, one should always look for a similarity solution, but in general one must resort to experiment or (even worse for some people) numerical methods. Fortunately for the more progressive amongst us, Barenblatt is very much in favour of the value of numerical experiments and emphasizes the great potential of computational science.

It turns out that this procedure – of using dimensional analysis where small dimensionless quantities are neglected – is not always successful but can be generalised. Thus an expression is sought, where the dimensionless expressions are now of the form

$$\Pi = \Pi_{l+1}^{\alpha_{l+1}} \dots \Pi_m^{\alpha_m} f\left(\frac{\Pi_1}{\Pi_{l+1}^{\beta_1} \dots \Pi_m^{\delta_1}}, \dots, \frac{\Pi_l}{\Pi_{l+1}^{\beta_l} \dots \Pi_m^{\delta_l}} \right)$$

and the exponents are to be determined by the requirement that a solution exists. This leads to nonlinear eigenvalue problems for the exponents. Barenblatt refers to this as *similarity of the second kind*. A detailed example of flame propagation is given by way of illustration.

The fourth chapter summarizes the results on the two types of similarity, ending with a useful guide to applying the methods and an amusing reference to a procedure to follow should one be unlucky enough to encounter a lion. Essentially, Barenblatt exhorts us not to give up when trying to solve hard problems.

A short chapter follows, where an explanation of the *renormalization group*, as used by theoretical physicists, is given. This is very useful, although one feels that the definitive guide to the relationship between renormalization, multiple scales, and the two forms of similarity has still to be researched and written.

A key idea explained in this book is that of *intermediate asymptotics*. Here, understanding is sought of a phenomenon at the stage where initial small-scale details are irrelevant but large-scale boundary effects are still to be felt. The analogy with painting is made: we stand far enough back for the brush strokes to be invisible, but close enough to appreciate the art. Scaling solutions are interpreted in this light, and are seen as deep results that lead to fundamental understanding. They are not just happy accidents.

The last three chapters provide further examples. The reduction of problems using the travelling wave ansatz is interpreted as a similarity solution. The scaling ideas involved in fractals and turbulence theory at very high Reynolds numbers are surveyed. In the earlier chapters Barenblatt showed how numerical results help the analyst to find good trial solutions, that turn out to be exact. In the final chapter he demonstrates the fundamental need for high quality experimental results and thus raises the status, yet further, of existing knowledge of turbulence discovered via dimensional analysis.

A closing paragraph, quoting Kolmogorov, captures the value of combining experiment and scaling. One can only conclude that any mathematical scientist could be inspired to fundamental advances in their own domain after studying this marvellous book.

CHRISTOPHER L. FARMER

Theory and Applications of Nonviscous Fluid Flows. By R. KH. ZEYTOUNIAN. Springer, 2002. 294 pp. ISBN 3540 41412 6. £57.50

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This book touches on a number of topics in fluid mechanics at an advanced level. From the title, it is evident that the starting point for the various applications is the set of Euler equations for inviscid flow.

After a discussion of the issues involved in deriving the compressible Euler equations from the Boltzmann equation (Chapter 1), and in deriving the Navier–Stokes equations from classical continuum theory (Chapter 2), the author gives a short presentation of the method of matched asymptotic expansions and the multiple scale method in Chapter 3. These methods are used on a number of occasions throughout the book. To illustrate the former method, a boundary-layer flow with a variable viscosity is analysed, and the latter method is applied to an investigation of one-dimensional, low Mach number, weakly nonlinear acoustic waves. Both applications are nicely developed and are essentially self-contained.

Various forms of the Euler equations are presented in Chapter 4, in some cases to get a head start on upcoming applications. The number of possibilities discussed is quite large so, in many cases, the equations are presented without prior development, e.g. Lilleys equation for acoustics, but references are abundant. Even Clebschs and Webers transformations are presented shades of Lamb and Truesdell.

Chapter 5 concerns applications in atmospheric flows and contains, for example, presentations of the quasi-geostrophic equations and the inviscid Boussinesq equations. A feature common to many of these sets of reduced equations, here and in other chapters, is their incompatibility with arbitrary initial data. The author clearly shows, using a matching procedure, how the initial adjustment takes place. Also included is the application of various approximate theories to lee waves.

Low Mach number flows and the incompressible limit are treated in a short Chapter 6. In particular, the multiple scale method is used to attack, in fair detail, the problem of acoustic waves in a bounded container. In Chapter 7 flow through turbomachines is considered where the reciprocal of the number of blades per row is considered a small parameter.

Shock waves and vortex sheets are considered in Chapter 8 as prominent examples in fluid mechanics of flows with surfaces of discontinuities. Included are a discussion of the internal structure of a shock and the authors theory of rolled-up vortex sheets. Finally, in Chapter 9, well-posedness and solvability of some Euler flow problems are considered along with existence and uniqueness of their solutions. No proofs are given but the discussions are extensive and ample references are given for the interested reader.

I believe the book would be a welcome addition to the bookshelf of anyone working in theoretical fluid mechanics. It would also be a valuable supplemental text for a post-masters course in fluid mechanics.

ANTHONY LEONARD

Introduction to Hydrodynamic Stability. By P. DRAZIN. Cambridge University Press, 2002. 276 pp. ISBN 0521 804272, £65.00 (hardback) or ISBN 0521 009650, £22.99 (paperback).

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The subject of hydrodynamic stability theory touches many areas of fluid mechanics. Essentially, it addresses the question of whether a given flow is likely to persist, or be replaced by some other flow. Its fundamental ideas shape our understanding of all sorts of physical phenomena, and can be easily illustrated using certain simple classic flows. In this manner, experimental observations concerning certain stages of the transition from laminar flow to turbulence can be explained. However, many aspects of this transition remain obscure for many flows, making this an important ongoing area of fundamental research. Also, there are major industrial and engineering flows, like aeronautical flows, where a better understanding of transition could have significant practical consequences. Therefore, hydrodynamic stability theory is likely to remain a central component of many PhD students' introduction to research (both theoretical and applied) in fluid mechanics, and should, perhaps, be regarded as essential material for the well-read PhD student of any branch of fluid mechanics.

Drazin's book is an excellent first introduction to this subject. It is especially well suited to a course on a taught postgraduate programme for general fluid mechanics students, containing about 120 problems (almost a third of the book) of a wide range of difficulty. Another desirable role for a graduate text is to form a bridge between undergraduate fluid mechanics and contemporary research. Drazin's book does not quite bridge this gap in some areas, though references are given to recent books that do, like to Schmid & Henningson's *Stability and Transition in Shear Flows* (Springer, 2001).

In reviewing this book, one is drawn inevitably into making comparisons with Drazin & Reid's *Hydrodynamic Stability* (Cambridge University Press, 1981), which is a standard text for the subject, and from which in many respects Drazin's book is derived. Drazin & Reid's monograph combined topics from Lin's *The Theory of Hydrodynamic Stability* (Cambridge University Press, 1955), which dealt largely with shear layers, with topics from Chandrasekhar's *Hydrodynamic and Hydromagnetic Stability* (Oxford University Press, 1961), which covered almost everything else, to produce a relatively complete survey of the subject at that time. However, since the early 1980s, the subject has developed in a number of ways. Examples include: the method of matched asymptotic expansions has replaced many of the historical techniques described in Drazin & Reid for obtaining analytical results at large Reynolds numbers; computational methods have become the most widely used of research tools; emphasis has also shifted from the 'method of normal modes' used throughout Drazin & Reid, to consideration of initial-value problems, both because of the transient growth that can occur even in stable systems at relatively short times, and because of the question of the propagation characteristics of initially locallized disturbances, i.e. whether a flow is convectively or absolutely unstable. Rather than treating these new areas, Drazin has sought to consolidate and simplify his treatment of the established core of the subject, making the material easier to read and generally much more accessible, and less intimidating, to today's graduate students. In this objective, he has succeeded admirably.

Subjects treated include bifurcations in model ordinary differential equations, Jeffery–Hamel flows (flow between non-parallel plates), Kelvin–Helmholtz instability (instability of a vortex sheet), capillary instability of a jet, Rayleigh–Bénard convection (thermal instability), centrifugal instabilities, piecewise-linear velocity profiles of inviscid flows and some qualitative descriptions of solutions of the Orr–Sommerfeld equation (i.e. stability of homogeneous parallel viscous shear layers). Subjects mentioned only briefly include absolute and convective instabilities, weakly nonlinear theory, non-parallel effects on the stability of boundary layers, cross-flow instabilities, the energy method, asymptotic methods and transient non-modal growth.

Although the opportunity to bring the subject fully up to date was not taken, the opportunity to write a clear straightforward introduction (as the title indicates) to the subject has been taken, and a very useful book has been produced that will be of interest to engineers, physicists and mathematicians starting research in fluid mechanics for many years to come.

J. J. HEALEY